|  |  |  |
| --- | --- | --- |
| **Prob #** | **Points** | **Max** |
| **1a** |  | **10** |
| **1b** |  | **15** |
| **2c** |  | **15** |
| **2d** |  | **15** |
| **2e** |  | **10** |
| **3** |  | **15** |
| **4** |  | **20** |
| **Total** | **0** | **100** |

1. A hollow fiber has a diameter of 200 m and a length of 10 cm. The nominal molecular weight cutoff for the pores (NMWCO) is 5,000 g/mole.
2. What is the pore radius in nm?
3. Find the actual sieving coefficient of a solute of molecular radius of for a fiber of this length and radius, but with a pore radius of .
4. For the same fiber in Part b, find the total plasma flux across the fiber wall, , assuming that the osmotic pressure is negligible and that the mechanical pressure across the wall is 20,000 Pa. Assume that , , pore tortuosity is 1.2, and the hollow fiber wall thickness is 60 m.
5. Find the filtrate concentration of a hypothetical molecule in this fiber if the actual sieving coefficient is 0.2, the total solvent flux is ml/s, the mass transfer coefficient is cm/s. Assume that the concentration of this drug is 0.02 M in the bulk solution. As in Part a, the fiber diameter is 200 m and its length is 10 cm
6. For the same hollow fiber in Part a, if the concentration of the filtrate is 10 mg/liter and the plasma flux per unit area across the wall is what is the rate of mass flux across the hollow fiber wall (in mg/s).
7. A hollow fiber has a mass transfer coefficient of . The fiber is coated with a drug on the inside walls with saturation concentration of 100 mg/L. Pure water enters at the inlet, while the drug exit concentration is 40 mg/L. Find the mass flux of the drug into
8. A spherical shell of radius is evenly coated on the inside with a drug whose saturation concentration is . Diffusion is in the radial () direction only and no reactions are present within the shell. No diffusion occurs outside of the shell. The fluid inside the sphere is not in motion. The situation is governed by Fick’s second law in spherical coordinates, and with diffusion in the radial direction, no velocities, and no reactions, this equation becomes

*The boundary conditions are saturation concentration at the sphere surface and zero derivative in concentration at the sphere center.*

*With initial condition*

Apply the separation of variables method to the partial differential equation and find the functional form for the radial and temporal dependence of . I.e., if , find the functions described by and . (Note: A list of differential equations and solutions that frequently arise from these 2nd order problems is provided at the end of this exam.) **You do not need to worry about using the boundary conditions to find the constants in these equations.**

**Potentially Useful Formulas**

# Solute Flux

# Nernst Equation

# Error Function

Some Frequently Encountered Ordinary Differential Equations

|  |  |  |
| --- | --- | --- |
|  | Equation | Solution |
| 1 |  |  |
| 2 |  |  |
| 3 |  | Real Roots,  Double Root,  Complex Roots  where |
| 4 | Fourier Equation: Special case of (3), where (no damping): | or |
| 5 | Similar to 4, but with negative linear term | or |
| 6 | Equidimensional Equation (Cauchy Euler Equation) | where are the roots of the equation obtained by substituting into the equation and solving for . |
| 7 | Bessel’s Equation |  |
| 8 | Or |  |
| 9 | Legendre’s Equation (canonical form) |  |
| 10 | Legendre’s Equation (alternative form) |  |
| 11 | Airy’s Equation |  |

**Fick’s First Law of Diffusion**

# Non-Dimensional Parameters

Let be a characteristic velocity, be a characteristic length, be kinematic viscosity,

Further, let be diffusion coefficient.

Let be the mass transfer coefficient.

# General Mass Transport Equation (Fick’s Second Law)

In Cardesian coordinates

In cylindrical coordinates

In spherical coordinages

# Boundary Layer Development

Pipe flow, fully developed momentum boundary layer .

Pipe flow, fully developed concentration boundary layer .

Flat plate, laminar boundary layer .

Cylinder, laminar flow .

# Constants

Avagadro’s Number:

Faraday’s Constant:

Universal Gas Constant:

Centigrade to Kelvin: Degrees Kelvin 273.15 Degrees Centigrade

**Table of Laplace Transforms**

|  |  |  |
| --- | --- | --- |
| 1 |  | 1 |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| 11 |  |  |
| 12 |  |  |
| 23 |  |  |
| 24 |  |  |
| 25 |  |  |
| 26 |  |  |
| 27 |  |  |

# Sherwood Numbers

|  |  |
| --- | --- |
| Condition | Sh |
| Sphere in a stagnant fluid | 2 |
| Forced convection over a sphere |  |
| Laminar flow over a flat plate |  |
| Laminar flow in a cylindrical tube, short contact time |  |
| Laminar flow in a cylindrical tube, fully developed flow and concentration profiles | 3.66 |
| Turbulent flow through a circular tube |  |
| Spinning Disk |  |
| Falling Film, Average |  |

# Oxygen Concentrations

Henry’s Law

Sphere

Planar Bioartificial Organ

Perfusion Bioreactor